

Extrapolation of traveltimes functions for 3D prestack depth migration

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Summary

The quality of depth images resulting from Kirchhoff prestack depth migration depends strongly on the ability of the traveltimes calculation algorithm to handle velocity variations. Ray-tracing techniques can handle rapid change in velocity field very well but are costly in terms of execution time. This paper demonstrates a way to decrease the computation cost by calculation of several traveltimes maps at once. The method is based on extrapolation of the 3 D traveltimes maps.

The algorithm is based on the wavefront construction technique to compute the traveltimes and ray-parameters given source and receiver locations. Assuming that the wavefront is locally planar then we may use only the ray parameter to extrapolate the traveltimes from one point to another one. This assumption is no longer valid for large values of the wavefront curvature. In order to extend the traveltimes extrapolation beyond this limitation, we exploit Dix's formula to express the local variations of the traveltimes. The algorithm proposed here performs the extrapolation of the traveltimes by using the first order approximation of the stacking velocity field, while the derivatives of which are calculated from the ray-parameter. The examples given here show that the extrapolation of traveltimes from one source to another can reduce the cost of the traveltimes computation by a factor of nine.

Introduction

The Finite Difference Eikonal (FDE) solver and the Ray Tracing (RT) are the most popular techniques used in Kirchhoff depth migration. The advantages of the ray-based techniques are the ability to handle strong lateral velocity variations and the possibility to access other information than traveltimes: amplitudes, ray parameters and caustics which are used in performing preserved amplitude depth imaging. Despite the cost of RT, which is higher than FDE solvers, these techniques are being introduced increasingly in production codes of Kirchhoff prestack depth migration. The selection of traveltimes values according to the amplitude or the ray-path (Nichols, 1998) allows better imaging of the complex structures.

The wavefront construction method (Vinje & al., 1996, Lucio & al., 1996) is well-accepted ray tracer algorithm. It propagates an optimal number of rays by generating an interpolated ray only if the paraxial prediction between rays fails (Fig. 1). The initial conditions of this new ray lead to the computation of the first derivative of the traveltimes at the source-point. Under the plane-wave assumption, the first order approximation may be used for the extrapolation

of the traveltimes. In the case of complex structures, this assumption is no longer valid.

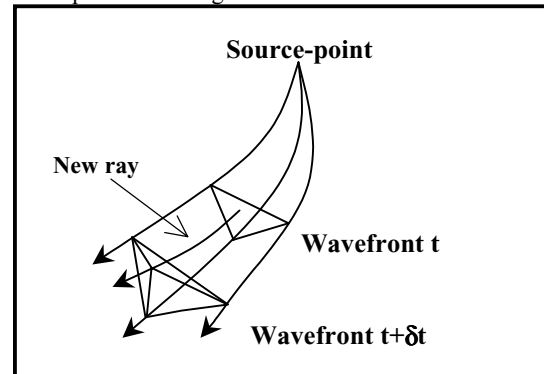


Fig.1: If the paraxial prediction between rays fails, a new ray is constructed and propagated.

The second order or the hyperbolic approximation of traveltimes (Schleicher & al. 1993) takes into account the wavefront curvatures. Several authors have used this approximation for different tasks. Pica (1997) proposed a new FDE solver based on celerity c ($c=d/t$, while t is the traveltimes and d is the distance source-receiver).

Gajewski (1998) used the hyperbolic approximation to compute the propagator matrix directly from the traveltimes maps.

Based on Dix's formula, Mendes (2000) proposes a new algorithm for the interpolation of traveltimes.

The important observation presented in these papers is that the celerity or its inverse varies more slowly than the traveltimes. The work present here is based on the same idea but it uses the ray-parameter in order to perform the variation of the celerity in an accurate way.

Method

Most of the 3D prestack Kirchhoff depth migrations algorithms compute the traveltimes from surface to depth first and then perform stacking along the diffraction surfaces. Each map of traveltimes is a function of five space variables: $S(x_s, y_s)$ and $M(x_m, y_m, z_m)$. Computing and storing in the traveltimes maps for all seismic trace positions and all imaging-grid points remains today an impossible task. Therefore, the traveltimes are computed for two sparse grids: the acquisition and the target grid (Fig. 2).

First, we compute the traveltimes maps for some of the nodes of the acquisition grid and then we extrapolate them to the other nodes.

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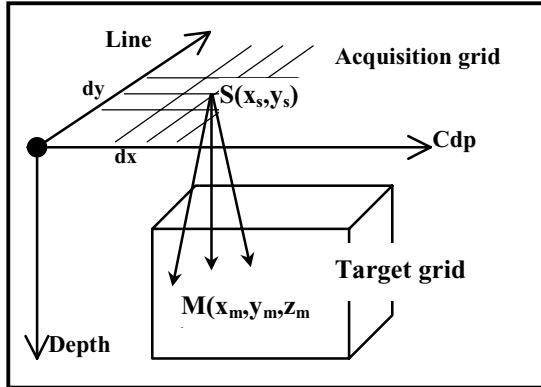


Fig. 2: The sparse grids used on computation of the traveltimes.

Traveltimes computation

Step one. We start by initializing the rays at the source-point S (about 500 rays) and parametrize them by the take-off angles θ and ϕ (Fig. 3). The eikonal equation $(\nabla t)^2 = s^2$ and the initial conditions θ , ϕ and $t=0$ allow an unique determination of the ray $\mathbf{r}^1(t, \theta, \phi)$ for any traveltme t . It can be represented as a set of t , position \mathbf{x} and ray-parameter \mathbf{p}

$$\vec{\mathbf{r}}(t, \theta, \phi) = (t; \vec{\mathbf{x}}; \vec{\mathbf{p}}).$$

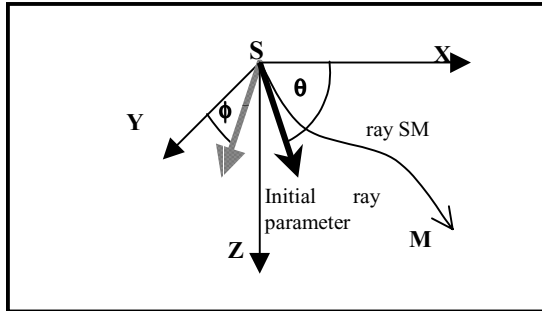


Fig. 3: The rays are parametrized by the take-off angles (θ and ϕ).

Step two. The propagation of a single ray is based on the following equation, which is solved by using fourth order Runge-Kutta scheme.

$$\frac{\partial \vec{\mathbf{x}}}{\partial t} = \frac{\vec{\mathbf{p}}}{s^2} \quad \text{and} \quad \frac{\partial \vec{\mathbf{p}}}{\partial t} = \frac{\nabla s}{s} \quad \text{Eq.1}$$

On the other hand, the paraxial ray tracing (Farra and Madariaga, 1987) allows the computing of the derivatives of the ray relate to θ and ϕ (Eq. 2). P is the propagator matrix given by:

$$\frac{\partial}{\partial(\theta, \phi)} \vec{\mathbf{r}}(t + \delta t, \theta, \phi) = P \frac{\partial}{\partial(\theta, \phi)} \vec{\mathbf{r}}(t, \theta, \phi) \quad \text{Eq.2}$$

Step three. In order to construct accurately the wavefront ($t + \delta t$), we check if two neighbor rays $\mathbf{r}(t, \theta_1, \phi_1)$ and $\mathbf{r}(t, \theta_2, \phi_2)$ may be mutually predicted. If this prediction fails then we add a new ray $\mathbf{r}(t, \theta', \phi')$ and propagate it from t to $t + \delta t$. In our case, the take-off angles of the new ray are computed as $\theta' = 0.5(\theta_1 + \theta_2)$ and $\phi' = 0.5(\phi_1 + \phi_2)$. These angles and the velocity at the source position give the initial ray-parameter. We will see later in this paper the way this vector is used in the extrapolation of traveltimes.

Step four. In order to compute the traveltimes at any point M of the target grid, the triangulated network is used to interpolate the traveltimes of rays propagating around M .

Step five. After computing the traveltme map $t(x_s, y_s, x_m, y_m, z_m)$ we can now extrapolate it at a new source point $S(x_s + dx, y_s + dy)$, where dx and dy are respectively the step of the acquisition grid along CDP and line. We express the traveltimes $t(x_s, y_s, x_m, y_m, z_m)$ by using Dix's formula

$$t^2(S, M) = \frac{[(t_0 V)^2 + (x_m - x_s)^2 + (y_m - y_s)^2]}{V^2} \quad \text{Eq.3}$$

and then express the unknown parameter t_0 and V (stacking velocity) as

$$(t_0 V)^2 = z_m^2$$

Finally, we obtain the following expression of the traveltme of the ray linking the source S to the depth-point M

$$t(S, M) = s(S, M)d(S, M) \quad \text{Eq.3.1}$$

where s is the stacking slowness and d is the distance SM . Note that Eq. 3.1 is similar to the fundamental equation used by Pica (1997) in the FD solver of the eikonal equation. Instead of computing the derivatives of the celerity by using the FD schemes in a sparse grid, we use the ray parameter at the source point, which leads to a more accurate computation of these derivatives. Actually, the horizontal components of the ray at the source point can be easily derived from the take-off angles and the velocity at this point

$$\begin{aligned} \frac{\partial t(S, M)}{\partial x_s} &= -\frac{\cos(\theta_m)}{V_s} \\ \text{and} \\ \frac{\partial t(S, M)}{\partial y_s} &= -\frac{\cos(\phi_m) \sin(\theta_m)}{V_s} \end{aligned} \quad \text{Eq.4}$$

where V_s is the velocity at S and θ_m and ϕ_m are the take-off angles of the ray SM .

Now, we can determine the derivatives of $s(S, M)$ with respect to x_s and y_s from the Eq. 3.1.

¹ Bold characters indicate vectors.

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$$\frac{\partial s(S, M)}{\partial x_s} = \frac{\partial t(S, M) / \partial x_s - s(S, M) \partial d(S, M) / \partial x_s}{d(S, M)}$$

and

$$\frac{\partial s(S, M)}{\partial y_s} = \frac{\partial t(S, M) / \partial y_s - s(S, M) \partial d(S, M) / \partial y_s}{d(S, M)} \quad \text{Eq. 5}$$

Assuming smooth variation of $s(S, M)$, the first order of the Taylor series may be used to determine $s(S+dS, M)$ at another source point of the acquisition grid

$$s(S + dS, M) \approx s(S, M) + \frac{\partial s(S, M)}{\partial x_s} dx + \frac{\partial s(S, M)}{\partial y_s} dy \quad \text{Eq. 6}$$

where $dS = (dx, dy)$.

Finally, the extrapolated traveltimes are obtained by introducing the extrapolated slowness (Eq. 6) into Eq. 3.1

$$t(S + dS, M) = s(S + dS, M) d(S + dS, M) \quad \text{Eq. 7}$$

The extra computation cost used to extrapolate traveltimes corresponds to the computation of the distance $d(S, M)$ and the derivatives of $s(S, M)$ only. Since this cost is negligible compared to the cost of the ray tracing, we can compute very fast 8 extra traveltimes maps

$$t(x_s \pm dx, y_s \pm dy, x_m, y_m, z_m).$$

Nevertheless, the limitations of this method depend strongly on the validity of Eq. 6. This equation is no longer valid for the cusp-points of the wavefront. Actually, we may no longer suppose the continuity of the stacking slowness or celerity at the points where the branches of the wavefront change.

In the case of multi-arrival events, we can select the most energetic arrival or the shortest ray-path. The extrapolation of amplitudes and ray-paths is also based on the same assumption used for calculation of the traveltimes. Therefore, the selection of traveltimes is no longer valid at the cusp-points of the wavefront.

Examples

In order to demonstrate the validity of the present method, we have calculated the traveltimes by using the wavefront construction in the 3 D SEG model. Line #340 of this model is shown in the Fig. 4. The shape of the salt-dome in the central part of the model presents difficulties in the accuracy the traveltimes required to image correctly the deeper reflector at 3 500 m.

The acquisition area has been discretized by taking the source positions every 4th Cdp and every 4th line, where the CDP and line interval being equal to 20 m. After tracing rays with an interval of 16 CDPs and 16 lines (320 m) we use the present method to compute the traveltimes at each point of the acquisition grid.

Fig. 5a displays the traveltimes map computed for the source position located at CDP #340 and line #340. The

traveltimes have been stored along line #341 and correspond to the most energetic arrivals. We have extrapolated this map from the source position located at CDP #344 and line #344 (Fig. 5b). By calculating the difference between two maps we can observe (Fig. 5c) that the error is less than 5 ms for the main part of the output grid. Nevertheless, there are two areas, shown by label A, where the difference exceeds 5 ms. Comparing to the previous maps, we can see that these areas correspond to the discontinuity of the branches of the wavefront. In this particular case, the tapering function used by the Kirchhoff algorithm (migration of the down going rays only) does not take into account these areas.

Conclusions

The extrapolation of traveltimes based on the derivatives of Dix's formula leads to a speed up of the computation with an acceptable error for Kirchhoff prestack depth migration. The algorithm we use for the extrapolation can also perform the compression of the traveltimes. Actually, we have shown that 3 maps only (traveltimes + ray-parameters) are needed to produce 9 traveltimes maps. Using a higher order approximation of the traveltimes, this algorithm can be extended to the extrapolation of the amplitudes and the ray-parameters.

References

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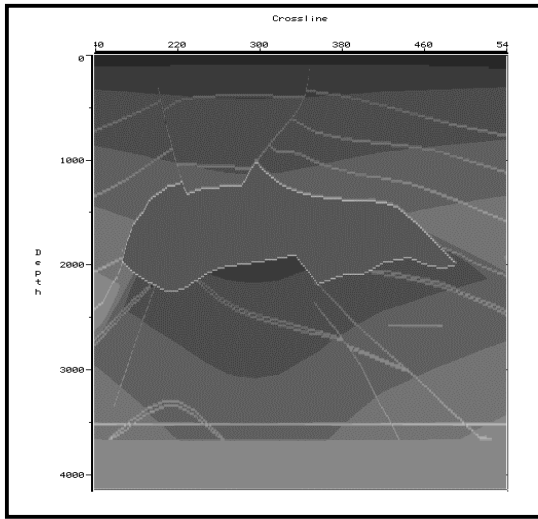


Fig. 4: Line # 340 of the velocity model (SEG 3D) used to compute the traveltimes.

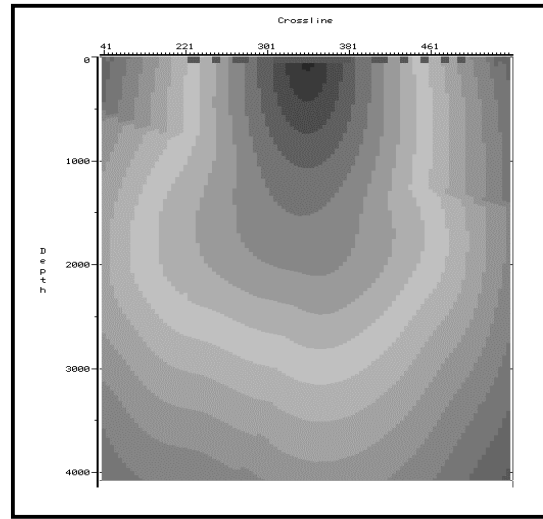


Fig. 5.b: The map of extrapolated traveltimes output at line # 340. They have been computed for the source position at Cdp # 344, line # 344 and extrapolated at Cdp # 340, line # 340. The distance between the source positions is 132 m.

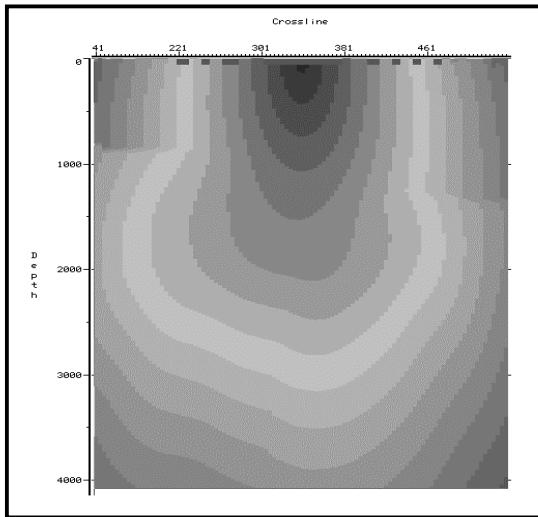


Fig. 5.a: The map of traveltimes output at line # 340. The coordinates of the source position are Cdp # 340 and line # 340.

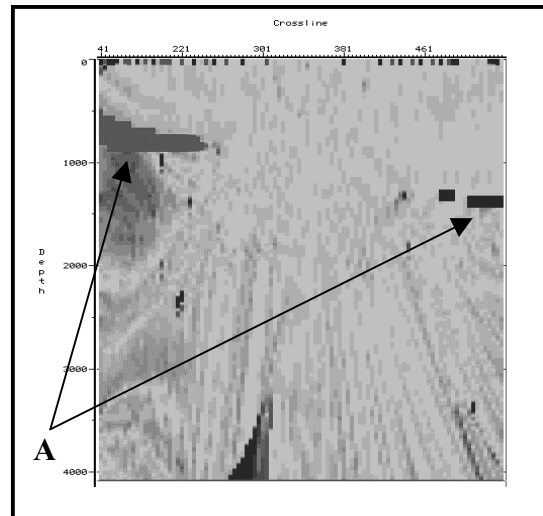


Fig. 5.c: The difference between the traveltimes maps (Fig. 5.a and Fig. 5.b). The displayed values have been clipped between -5 and 5 ms. The label A shows the areas where the differences between the computed and extrapolated traveltimes exceed 5 ms. That is due to the criterion we used to select the traveltimes, the most energetic.