Comparison of Kirchhoff and reverse-time migration methods with applications to prestack depth imaging of complex structures

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Summary

The performance of the two popular migration methods, the Kirchhoff integral and the reverse-time migrations, are evaluated through applications to imaging complex structures using prestack shot records. The migration results from the Marmousi model data demonstrate that the reverse-time migration is more accurate in imaging the steeply dipping faults. Its better accuracy is based on the use of the correct velocity model, and is paid off by its huge amount of computations. In the application to the Alberta Foothills data where a good estimate of the velocity model is available; however, both the Kirchhoff and the reverse-time migration methods produce almost identical results. This implies that in the real world of exploration seismology it will be relatively difficult to identify which method performs better as we will never know the exact answer of the subsurface.

Introduction

Migration is the processing step of constructing the true subsurface structure from the recorded seismic data. Because of its significance in interpretation, many advanced migration methods have been proposed in the past couple of decades. Kirchhoff integral migration (Schneider, 1978, Berkhout, 1982) and reverse-time migration (Baysal et al., 1983, McMechan, 1983) are the most popular methods. Both methods are soundly based on the wave equation, the mathematical description of seismic wave phenomena. Theoretically they both are capable of migrating steep dip reflections. Both Kirchhoff and reverse-time migrations have been applied to real seismic data. They have seen success here or there. Lamer and Hatton (1990) give a very objective comparison of Kirchhoff integral and finite-difference migrations in the case of stacked data with the conclusion that both methods produce comparable migration accuracy, although their finite-difference migration is based on one-way wave equations. Whitmore at el.(l988) obtain similar conclusions when they do a comprehensive survey of depth migration methods on stacked data. Here, we intend to evaluate the two methods with applications to prestack depth imaging of complex geologies.

Theory

Kirchhoff migration can be performed both recursively and nonrecursively. Our choice of the nonrecursive fashion is largely based on our ability to accurately calculate the wavefront traveltimes. This eliminates the need of extrapolating the wavefields from depth to depth without sacrifice of accuracy. Generally, for a model of N_x by N_z grid points, migrating one shot with N traces using the Kirchhoff integral method will take $O(N'_x \cdot N_z \cdot N)$ operations where N'_x is the migration aperture. It is seen from this expression that the computation is directly dependent on the number of traces in the gather, so the computation for a gather of less traces will take less time. The computation of traveltimes takes about 40% of the total computations. This number is somewhat dependent on the complicated nature of the model which arises from the search of computing wavefronts in traveltime calculation. However, the number can be much reduced by setting up time tables before the integration procedure. In addition, the Kirchhoff migration used here is accurate to the, extent that both the far-field approximation and the negligence of the obliquity are acceptable. This presumption certainly is in error in the near surface of the earth. One of the most important attributes of Kirchhoff method perhaps is that it can use selective shots and traces to image some prespecified targets as it is trace based. This also makes the Kirchhoff method easy to use in areas with rough topography. Thus, static corrections can be easily contained in the Kirchhoff shot migration. Furthermore, the preparation of model and data in Kirchhoff migration is much simpler than other methods. The selectivity of the data, high computation efficiency, plus the easy preparation of data sets render Kirchhoff migration to be the preferred method for use, especially in the process of recursive migration and velocity analysis.

Reverse-time migration is recursive in time. It is a very accurate method as the only possible error is the discretization error when differentials are approximated by finite differences. Its high accuracy is nevertheless traded off by its very intensive computations. For a model of N_x by N_z, reverse-time migrating a single shot of N traces with each consisting of N_t samples, will take $O(N'_x \cdot N_z \cdot N'_t)$ operations where N'_t is the extrapolation time steps. Compared to the $O(N'_x \cdot N_z \cdot N)$ operations involved in the Kirchhoff scheme, reverse-time migration will generally require much more computations, as N'_t, would be much larger than N in most cases. It is apparent from this estimation that the computations involved in reverse-time migration are independent of the number of traces in each shot, which is definitely in contrast to Kirchhoff method. So reverse-time migration for a gather of a single trace is computationally just the same as migration of a gather with many traces. This estimation also implies that when grid size is halved for a given model the computation time will increase to 8 times of the original for the reverse-time migration, while 4 times of the original for the Kirchhoff method. Moreover, the preprocessing for reverse-time migration used to be considered as being a bit more complicated too. Our recent study, however, indicates that interpolation of missing traces, which is very difficult in complicated areas, can be bypassed in many cases as wavefields are capable of healing themselves during the reverse-time extrapolation procedure. Despite the high demand of computations, reverse-time migration tends to have more wide applications. This is due to the recent advance in computer sciences, and the preferred high accuracy of the method. Compared to the Kirchhoff method, its independence of accuracy and computations on the complexity of the geological model is also an advantage. These characteristics, in addition to its implicit ability of static corrections, filtering, and self-healing of wavefields make reverse-time migration a very powerful method for imaging geologically complex areas.

Data examples

In this section, we will show the performance of both the Kirchhoff and the reverse-time migrations on two data sets. The first example is the well-known Marmousi model data. The model data present a challenge to exploration geophysicists in imaging complicated geological provinces (Versteeg, 1993). The model contains very complicated geological features, especially the shallow steep faults and the underlying high velocity salt body intrusion. It has served as a standard test data set for both migration, inversion algorithms, and velocity analysis methods (Versteeg, 1993, Gray and May, 1994, Nichols, 1996). Figure 1 shows the migration image with the Kirchhoff integral when a 12.5 m grid size is used. It takes about 2.44 hours of CPU time on Memorial University 's campus computer AlphaServer 1000 with a clock frequency of 200 MHz. Figure 2, on the other hand, shows the stacked migration section with the reverse-time migration algorithm using the same gridded velocity model. However, this migration takes 21.43 hours of CPU on the same machine. These two plots are displayed with the same plotting parameters, so a direct comparison should be applicable. It is apparent that both methods have fairly well restored the geological features of the model. Nevertheless, as we notice, there are several places where the two images are different. In particular, the left and the middle faults in the Kirchhoff result is not as sharply defined as in the reverse-time migration image. These differences are mainly due to the algorithm details involved in the two methods, especially the negligence of the obliquity and the use of first arrival times in our Kirchhoff algorithm (Nichols, 1996). Thus, the reverse-time migration gives a more accurate migration image than the Kirchhoff method. Its higher accuracy is nevertheless based on the use of the actual velocity model, and is paid off by the much higher amount of computations.

The second example is the Husky Foothills data as used in the 1995 SEG migration workshop. This data set is anticipated to serve as an excellent real test data for imaging complicated structures. The Canadian Foothills is characterised by overthrust structures of great variety. Generally in these mountainous regions, there are acquisition problems. Nevertheless, the distributed foothills line is of excellent signal quality. Figure 3 shows the Kirchhoff migration result when a 10 m by 10 m gridded velocity model is used. The velocity model is initially set up based on structural geological information and the stacked section. The well logging information nearby provides good constraints to the velocity model. The model is then updated iteratively by prestack depth migration, migration velocity analysis, and geological interpretation. In this migration image, the shallow dipping formations at the upper left side of the section are clearly seen to be detached on the floor thrust which is at about the depth of 2600 m. Two other thrust faults are also well defined around CDP numbers of 580 and 810 respectively. Overall, this migration result offers a very encouraging result which is relatively easy to be interpreted. The migration of this foothills line, however, only takes about 22.91 hours of CPU time. In fact, in our early stages of studying this line, the migration is done on a much coarser grid which is 20 m by 20 m. In that case, the Kirchhoff migration takes about 5.5 1 hours of CPU time with quite similar results. From the CPU times, it is clear that use of a twice fme grid will increase the CPU time to about 4.2 times, which is pretty close to our theoretical estimate of 4 times, considering the overhead of computations involved in the migration. In contrast, Figure 4 shows the prestack migration section with reverse-time migration. It is based on the same velocity model as used in Figure 3. It essentially reveals the same salient features of the geological structure as Figure 3. However, the production of this image requires about 135.5 5 hours of CPU time! This is definitely a very big amount of computer time compared to that taken by the Kirchhoff method. The similarity of the migration results between the Kirchhoff and the reverse-time migration method does not indicate that our Kirchhoff method is as accurate as the reverse-time method. Nevertheless, it only implies that there are still errors in the velocity model. Due to these errors, it is not clear which method works better in achieving migration accuracy. This possibly shows that in the real world of exploration seismology, where only an approximation of the true geological and velocity model is available, even the approximate version of Kirchhoff method may work as well as the accurate reverse-time migration.

Conclusions

Prestack depth migration is a viable means for imaging complex geological scenarios. Kirchhoff and reverse-time migrations are the most popular methods using prestack seismic data to image such complex structures. Both methods are based on the wave equation, and theoretically have no limit in migrating steep dips. In the sense of prestack migration, either one can be directly applied to areas with rough topography. They share the success of being practically applied to real data.

The application of the two methods on the Marmousi model data strongly supports that reverse-time migration does a better job of imaging the steeply dipping faults. This possibly is due to our negligence of the obliquity factor and the use of first break times in the Kirchhoff integral. This higher accuracy of the reverse-time migration is nevertheless based on its use of the correct velocity model. and is traded off by its large amount of computations. In conhart, the migration images of the Husky Foothills data. separately obtained by the two methods, are very similar. To this real data ret, even though we have a pretty good estimate of the model, it is still quite difficult to differentiate one result from the other. The similarity of the results suggests that the velocity model is still in error. It is probably these errors which make the migration results ambiguous.

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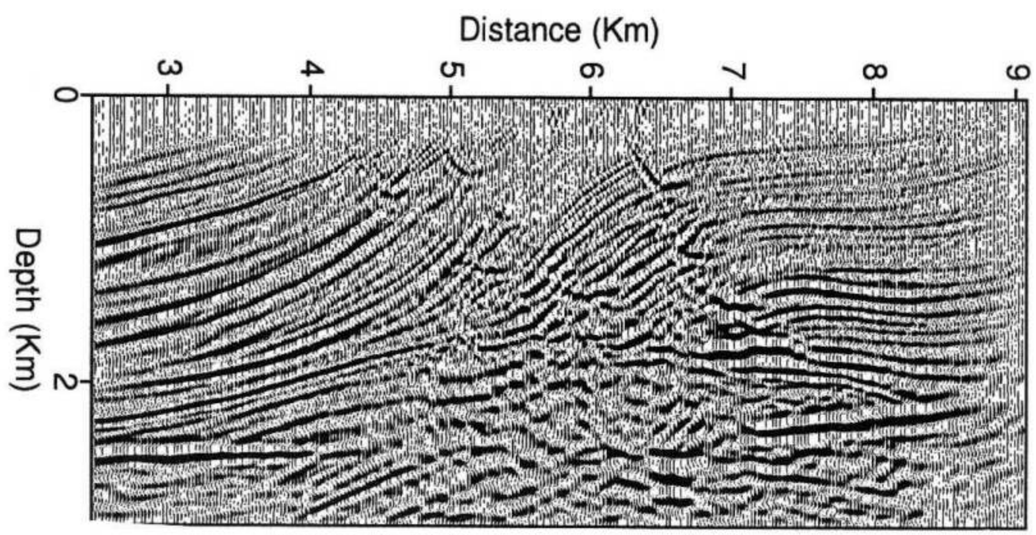

Figure 1. Kirchhoff migration section of the Marmousi data.

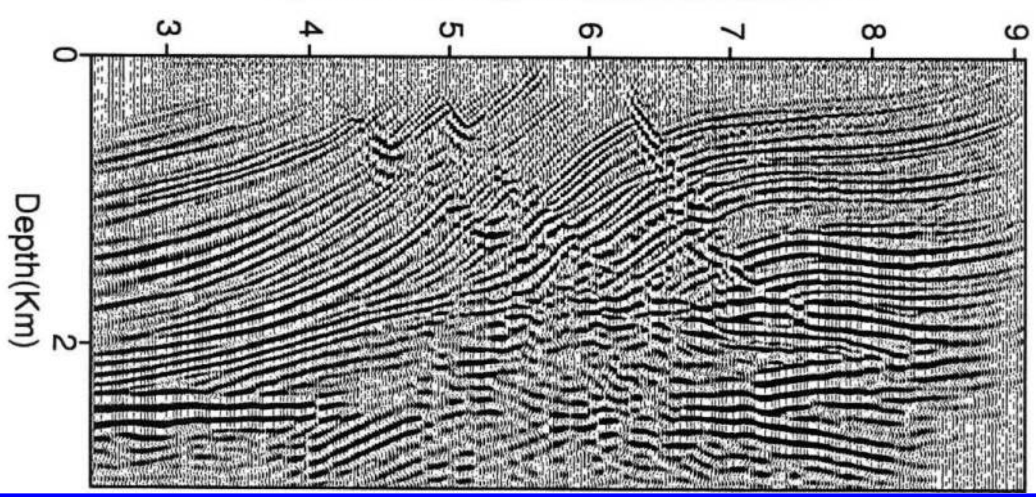

Figure 2. Reverse-time migration section of the Marmousi data

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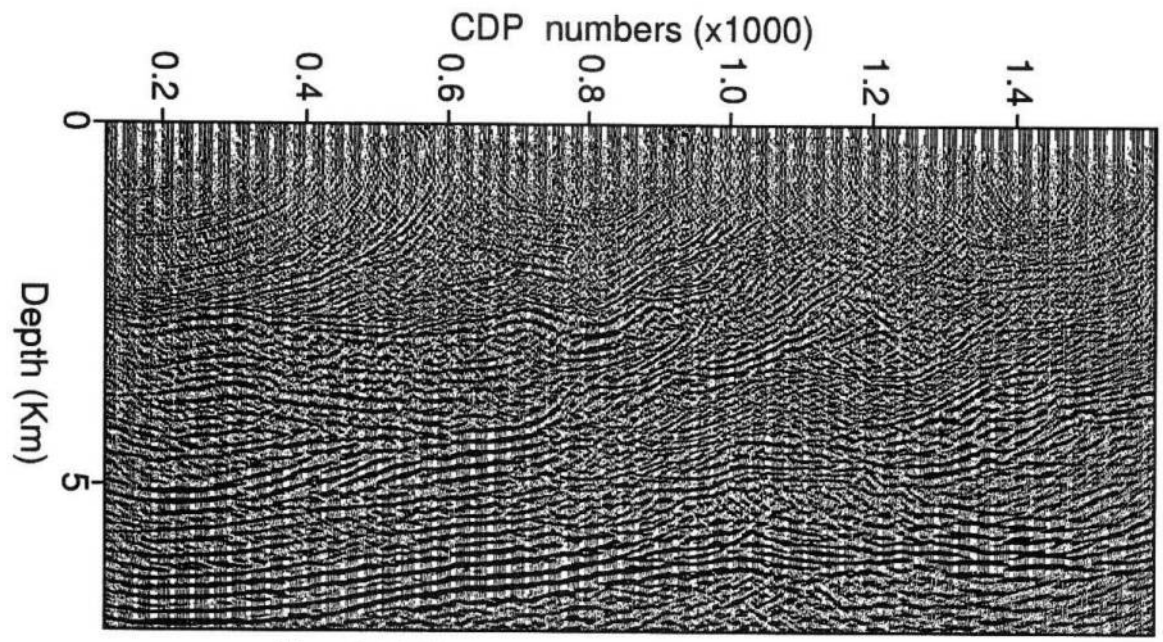

Figure 3. Kirchhoff migration image of the Husky Foothills line.

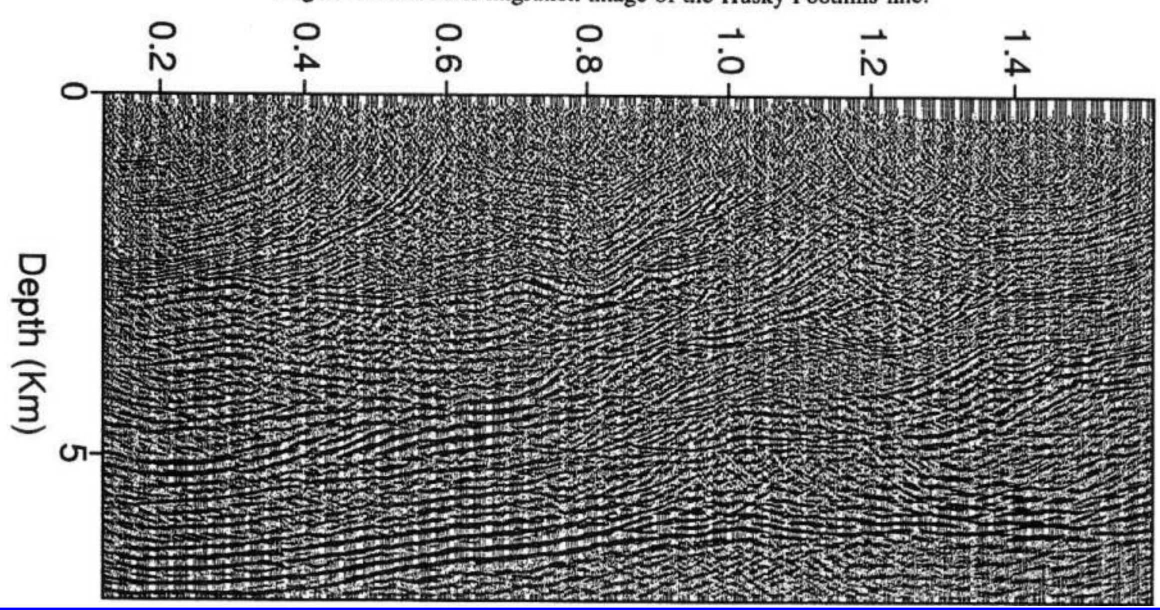

Figure 4. Reverse-time *migration* image of the Husky Foothills line.