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Discussion and Reply

On "Angle-domain common-image gathers by wavefield continuation methods" (Paul C. Sava and Sergey Fomel, 2003, GEOPHYSICS, 68, 1065–1074)

Discussion by Allon Bartana[1], Dan Kosloff[2], and Igor Ravve[1]

Sava and Fomel (2003) present an original approach for the generation of angle-domain common-image gathers (CIGs) by wave-equation migration. Their procedure consists of two steps. First, local-offset gathers are created during the migration. Second, depth-migrated angle gathers are generated by applying a slant stack to the local-offset gathers.

Angle-domain gathers are very important for velocity analysis and amplitude versus offset (AVO) studies. Unfortunately, we believe that the procedure outlined in the article does not yield correct angle gathers when there are velocity errors. In the following, this is demonstrated for the simplest example of a single horizontal reflector embedded in a uniform-velocity medium.

Angle moveout for a single horizontal reflector

Consider a single horizontal reflector at depth z_0 embedded in a uniform velocity medium with velocity c (Figure 1). Since this problem is spatially invariant in the horizontal direction, the traveltime as a function of offset in each of the CMP gathers is given by

$$t^2 = t_0^2 + \frac{x^2}{c^2}, \quad (1)$$

where x is the surface offset and $t_0 = 2z_0/c$ is the zero-offset traveltime.

When the CMP gathers are migrated with an incorrect velocity v, each offset produces a reflector image at an erroneous depth z and corresponding zero-offset time $t'_0 = 2z/v$ (Figure 1), where

$$t'^2_0 = t^2 - \frac{x^2}{v^2}. \quad (2)$$

Combining equations 1 and 2, we obtain

$$t'^2_0 = t_0^2 + \frac{v^2 - c^2}{v^2 c^2} x^2. \quad (3)$$

Substituting $t'_0 = 2z/v$ and $x = 2z \tan\alpha$ (Figure 1), we obtain the reflection-angle move-out equation,

$$z = \frac{v t_0}{2\sqrt{1 - \left(\frac{v^2}{c^2} - 1\right)\tan^2\alpha}}. \quad (4)$$

Figure 2 presents a vertical portion of an angle gather from Kirchhoff-angle migration (Koren et al., 2002), obtained for a synthetic model with parameters $c = 2000$ m/s, $v = 2200$ m/s, and $t_0 = 1$ s. Also shown in the figure is the theoretical $z(h)$ curve according to equation 4. Distances are measured in meters and angles are in degrees.

Kinematics of local-offset gathers for horizontal reflectors with uniform velocity

Let $P(x_m, h, z, t)$ represent the downward-continued wavefield in a shot-geophone 2D prestack depth migration; x_m is the midpoint coordinate, h is half of the local offset, and t is time. For a given CRP location x_m, the local-offset gather $g(h, z)$ is obtained according to

$$g(h, z) = P(x_m, h, z, t = 0). \quad (5)$$

Figure 3 shows the geometry of a reflection event on a local-offset gather for a horizontal reflector in a uniform medium. One can verify from the figure and equation 1 that

$$z = v\sqrt{\frac{t^2}{4} - \frac{\left(\frac{x}{2} - h\right)^2}{v^2}} = v\sqrt{\frac{t_0^2}{4} + \frac{x^2}{4c^2} - \frac{\left(\frac{x}{2} - h\right)^2}{v^2}}. \quad (6)$$

An image is formed when the depth z is stationary with respect to change in offset. Setting dz/dx to zero in equation 6 yields a relationship between the surface offset x and half-offset h:

$$x = -\frac{2h\,c^2}{v^2 - c^2}. \quad (7)$$

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Substituting equation 7 into equation 6 yields

$$z = v\sqrt{\frac{t_0^2}{4} - \frac{h^2}{v^2 - c^2}}. \qquad (8)$$

This equation relates the depth of a reflection event on a local-offset gather to the local offset.

Figure 4 shows a vertical portion of a local-offset gather obtained from shot-geophone wave equation prestack depth migration with the same model parameters as in the previous section. Also shown in Figure 4 is the predicted $z(h)$ curve according to equation 8. As the figure demonstrates, the match is very good.

Kinematics of reflection-angle gathers

In the procedure outlined by Sava and Fomel (2003), angle gathers are created from the local-offset gathers $g(h, z)$ by a

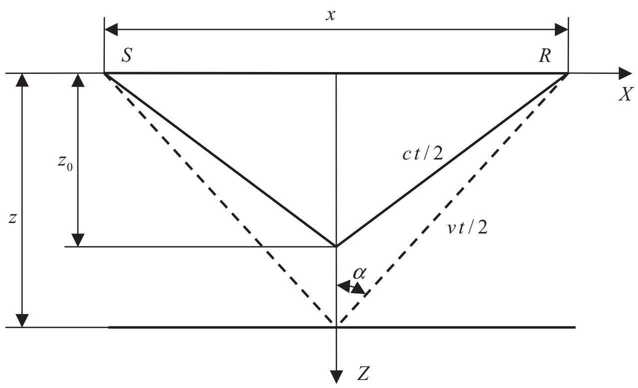

Figure 1. Horizontal subsurface model and raypaths for the correct velocity and for the image after migration with an overestimated velocity.

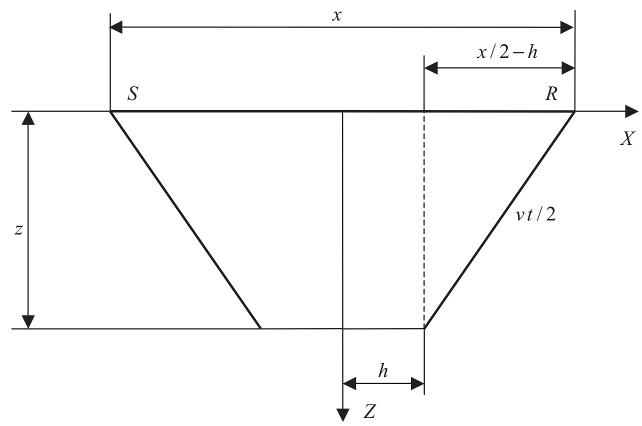

Figure 3. Geometrical configuration in a horizontal velocity model for a local-offset gather.

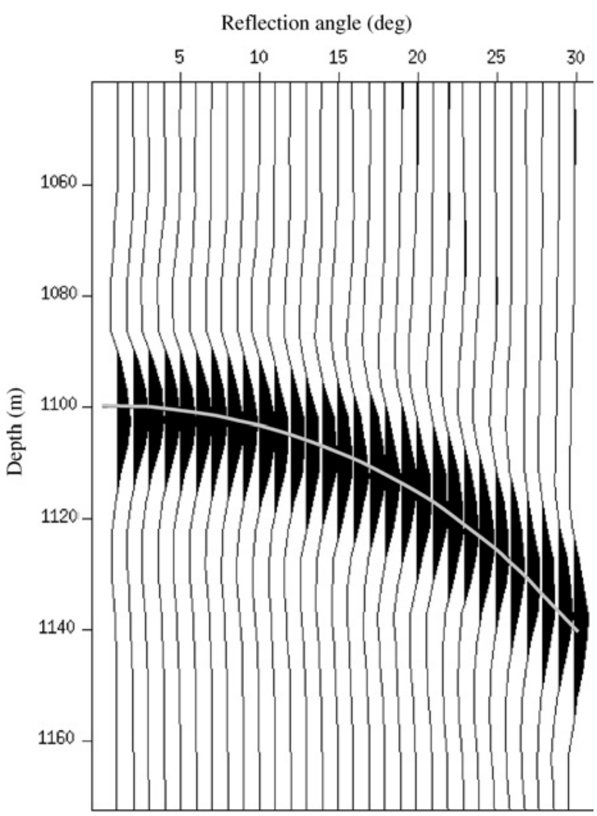

Figure 2. Depth-migrated gathers from angle-domain Kirchhoff migration and comparison with the theoretical moveout curve (solid line).

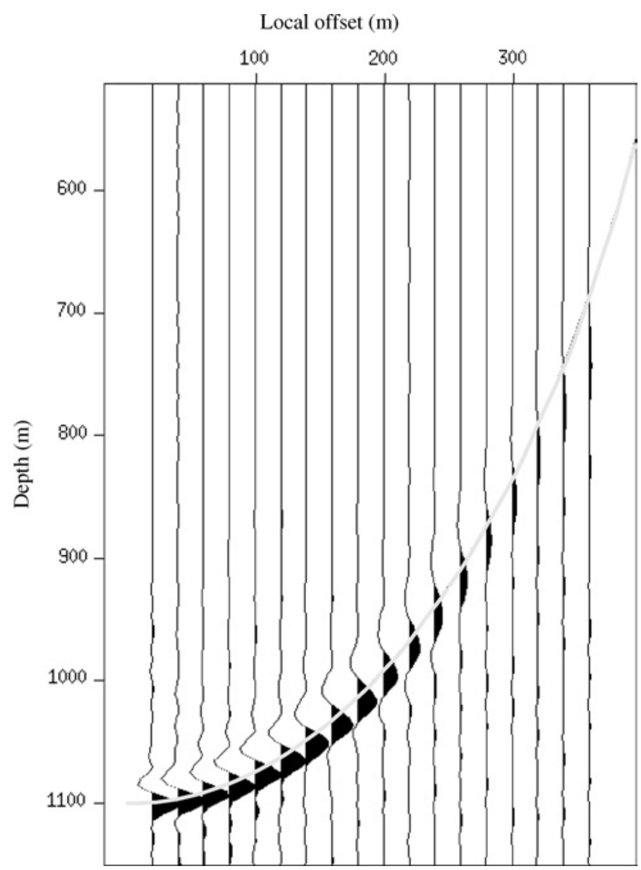

Figure 4. Local-offset gather from shot-geophone wave-equation migration and comparison with the theoretical moveout curve (solid line).

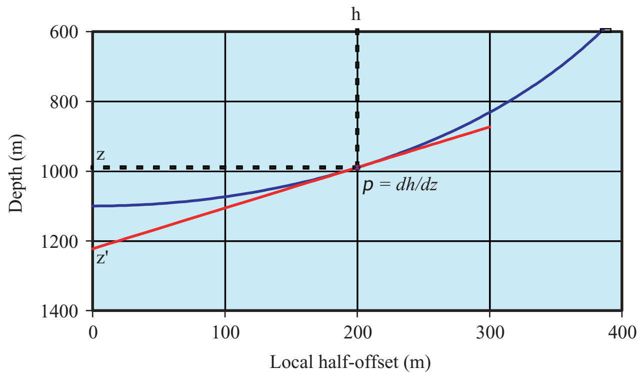

Figure 5. Geometrical configuration for the production of angle gathers from local-offset gathers.

slant-stack operation. This can be written

$$\bar{g}(p, z') = \sum_h g(h, z' - ph). \tag{9}$$

They claim that $p = \tan\alpha$, where α is the reflection angle.

Figure 5 shows the geometry for producing the slant-stack mapping. An event (z, h) on a local-offset gather is mapped to slope and depth (z', p) according to

$$z' = z - hp. \tag{10}$$

For the horizontal reflector model and a given value of h, z is obtained from equation 8 and p is obtained by taking the derivative of this equation with respect to h. This allows us to predict the move out curve $z'\,[\arctan(p)]$ on a migrated angle gather.

Figure 6 shows a vertical portion of a depth-migrated angle gather for the previous model parameters. Also shown in the figure are the predicted $z'\,[\arctan(p)]$ curve according to the procedure of Sava and Fomel as well as the $z(\alpha)$ curve according to equation 4. As Figure 6 shows, these two curves deviate quite noticeably. The depths of the migrated gather match the depths according to equation 10 and not the correct depth curve according to equation 4.

Therefore, the procedure of Sava and Fomel does not produce correct reflection-angle gathers in the presence of velocity errors. However, these gathers may give a good indication of the direction of the velocity errors (e.g., events curve upward for too low a velocity and curve downward for too high a velocity).

Sava and Fomel suggest a second procedure to generate slowness gathers $p_h = k_h/\omega$ during migration. Although these

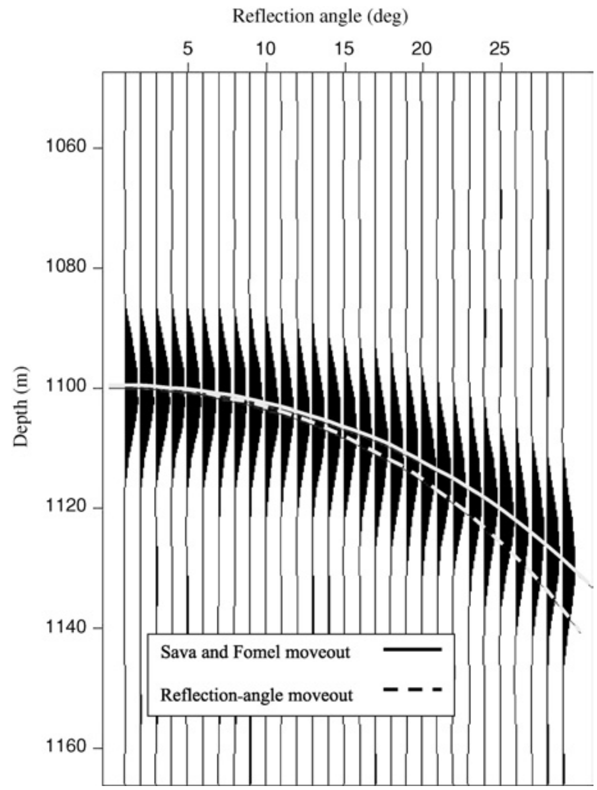

Figure 6. A $\bar{g}(p, z')$ gather obtained by wave-equation shot-geophone migration and comparison with analytical move-out by Sava and Fomel (solid line) and the theoretical move-out curve of Figure 2 (dashed line).

gathers do not give the reflection angle directly, we recommend using them with velocity-determination methods that require precise moveout, such as depth tomography.

ACKNOWLEDGMENTS

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Reply to the discussion

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We thank Bartana et al. for their interest in our paper and for suggesting an interesting topic for discussion.

In our paper (Sava and Fomel, 2003), we do not address the issue of using wave-equation angle gathers for velocity

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