DISCUSSION

On: "Accurate depth migration by a generalized phase-shift method" by Dan Kosloff and David Kessler (GEOPHYSICS 52, 1074-1084, August 1987).

I would like to point out that the "generalized phase-shift method" (Kosloff and Kessler, 1987) introduced for solution continuation — and referred to as a new method — is essentially identical to the "generalized Haskell matrix/layer eigenstate propagator (GHM/LEP)" method I presented earlier (Pai, 1985). Unfortunately, Kosloff and Kessler did not reference that paper.

To facilitate discussion, I first summarize the GHM/LEP method. In the method, an arbitrarily varying medium is treated as a stack of laterally inhomogeneous but vertically homogeneous layers. Within a layer, the wave equation is decoupled to an eigenvalue equation in x (the horizontal coordinate) defining a set of orthogonal eigenstates [see equations (4) and (5)] and a wave equation in z (the vertical coordinate) showing the eigenstates as upgoing or downgoing waves [see equations (6) and (7)]. In this way, the solution within each layer is a summation of $2N$ modes: N upgoing and N downgoing eigenstates. A solution propagator — a $2N \times 2N$ matrix — is constructed for solution continuation from one depth to another. The solution propagator is in terms of layer propagators, which are diagonal matrices describing mode propagation within layers as shown in equation (8), and interface propagators, which are nondiagonal matrices describing mode couplings at layer interfaces as shown in equations (10) to (14). The solution propagator can be used in a number of ways. For modeling, the solution propagator is used to construct the response to excitations, source or incident wave excitations, subject to the required boundary conditions — see equations (15), (21), and (22). For migration, the solution propagator can be used for surface data downward continuation. The GHM/LEP method is a solution method for field equations in general, not at all restricted to wave equations; the latest application has been to the diffusion equation in borehole electromagnetics (Pai and Huang, 1988).

The name "layer eigenstate propagator" is descriptive of the method itself. The name "generalized Haskell matrix" comes from the fact that when the medium is laterally homogeneous, all layers have the same set of eigenstates — namely, e^{ikx} (or k waves) — and the $2N \times 2N$ solution propagator is decoupled to N separate 2×2 matrices (one 2×2 matrix per k wave) which are just the well-known Haskell propagator matrices.

As summarized in the following, except for a few trivial variations, Kosloff and Kessler's derivation on generalizing the phase-shift method through eigenstate expansion [equations (1) to (17)] is essentially identical to the GHM/LEP method:

1) As in my paper, Kosloff and Kessler treat the medium as a stack of laterally inhomogeneous but vertically homogeneous layers, as indicated by the sentence just before their equation (7).

2) As in my paper, Kosloff and Kessler define a set of eigenstates in terms of an eigenvalue equation in x. In defining eigenstates, their equation (8) is equivalent to my equation (4), other than two trivial variations. First, my matrix defining eigenstates is $N \times N$ [equation (4) in my paper], whereas their matrix is $2N \times 2N$ [equation (8) in their paper], since they deal with P and $\partial P/\partial z$ (their nomenclature) simultaneously. Their choice is unnecessarily complicated: the actual dimensionality is only N, since the same eigenstates can span P as well as $\partial P/\partial z$ as indicated by their own expression of eigenvectors, each of which consists of two identical N-dimensional vectors [see the expression following equation (7) in their paper]. Second, in defining eigenstates, I went from the x-space to the k-space and then to the eigenspace in order to show that the k-space happens to be the eigenspace when the medium is laterally homogeneous (thus showing the generalization of the Haskell matrix method), whereas they went directly from the x-space to the eigenspace [and subsequently needed an extra page — equations (13) to (17) — to show the connection to the k-space]. Because of these two variations, their equation (8) may look different from my equation (4), but the two equations are in fact equivalent.

3) As in my paper, Kosloff and Kessler transform their solution into the eigenstate, up-and-down wave representation [equation (11) in their paper, with $\underset{\sim}{\mathbf{Q}}^{-1}$ as the transformation matrix] to obtain the diagonal propagator matrix [equation (12) in their paper]. Their $\underset{\sim}{\mathbf{Q}}^{-1}$ is my $\underset{\sim}{\mathbf{M}}$ in equation (5) combined with my equation (9) relating the up-and-down wave representation to the Ψ, $\partial\Psi/\partial z$ (my nomenclature) representation. Incidentally, half of $\underset{\sim}{\mathbf{Q}}^{-1}$ in their equation (10) is missing.

4) Kosloff and Kessler's final result (the part on generalization through eigenstate expansion), equations (11) and (12) showing the diagonal propagator of the eigenstate modes within a layer, is just my equation (8) defining the layer propagator. The elements of this diagonal matrix are all phase-shift factors just like those of the ordinary phase-shift method. This result establishes connections with and generalizes the ordinary phase-shift method.

In summary, Kosloff and Kessler's generalization of the phase-shift method through eigenstate expansion is identical to the GHM/LEP method I gave earlier, namely: *the horizontal-vertical decoupling of the wave equation through a solution expansion in terms of generalized eigenstates and subsequent constructions of propagators in terms of these eigenstates for solution continuation.*

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References

Pai, D. M., 1985, A new solution method for wave equations in inhomo-

geneous media: Geophysics, **50**, 1541-1547.
Pai, D. M., and Huang, M., 1988, A generalized Haskell matrix method for borehole electromagnetics: theory and applications: Geophysics, **53**, 1577-1586.

Reply by the authors to D. M. Pai

We regret not having referenced the work of Pai (1985), but we disagree completely with the contents of his discussion. Briefly summarized, the downward continuation in our article includes the following steps:

A) Write the governing equations as a system of first-order ordinary differential equations in depth of the form

$$\frac{d\mathbf{V}}{dz} = \underset{\sim}{\mathbf{A}}\mathbf{V}\,, \tag{1}$$

where **V** represents the quantities to be downward-continued (pressure and vertical pressure derivatives in the case of full acoustic migration), and $\underset{\sim}{\mathbf{A}}$ is a spatial operator.

B) Write the formal solution to equation (1) as

$$\mathbf{V}_{z+dz} = e^{\underset{\sim}{\mathbf{A}}dz}\mathbf{V}_z\,, \tag{2}$$

where dz is the depth increment. This equation in our opinion defines the generalized phase shift method.

C) Expand the formal solution by the Chebychev series

$$\mathbf{V}_{z+dz} = \sum_k C_k\, J_k(R)\; T_k\left(\frac{\underset{\sim}{\mathbf{A}}dz}{R}\right)\mathbf{V}_z\,, \tag{3}$$

where $C_k = 1$ and $C_k = 2$ for $k \neq 0, J_k$ are Bessel functions, and T_k are modified vector Chebychev polynomials. R is a number larger than the largest of the eigenvalues of $\underset{\sim}{\mathbf{A}}dz$.

Our migration technique is not based on an eigenfunction expansion. We believe that methods based on such expansions (like Pai's suggestion) would be prohibitively expensive. Morever, the whole purpose of our article is to show how to avoid this operation while maintaining high accuracy. The eigenfunction expansion option was brought up as background material to show the relation with the ordinary phase-shift method in the case of horizontally uniform structures, as well as to discuss the role of evanescent energy. In fact, as an Associate Editor of GEOPHYSICS suggested, a large portion of the first 4 pages of our article can be omitted without seriously affecting the readability.

In summary, we do not see a similarity between the method in our article and the methods suggested in Pai's work.

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Response by D. M. Pai to the Reply by the authors

In consideration of the authors' reply, I must clarify the following: This discussion is solely concerned with pointing out that the eigenstate expansion method in the first 4 pages of their paper [the material encompassing equations (1) to (17)] is essentially identical to my GHM/LEP method; this discussion is not at all concerned with the Chebychev expansion method in the latter part of their paper [the material following equation (18)], nor is this discussion concerned with advocating one method over another. Thus the authors' reply that their method is the Chebychev series expansion method with no similarity to the GHM/LEP method does not wholly bear on the discussion. In regard to their reply that the eigenstate expansion method serves merely as background material with little bearing on the central theme of their paper, I would like to point out the following: In their paper, Kosloff and Kessler presented the eigenstate expansion method and named it (and the title of their paper) the generalized phase-shift method [see the major heading "the generalized phase-shift method" encompassing the material from equation (1) to (12); see also, for example, the paragraph just above that heading on describing what they meant by the generalized phase-shift method]. Thus the eigenstate expansion method was in fact the foundation behind the title of their whole paper. Because of the prominence and significance attached to the eigenstate expansion method in their paper, I deem it necessary to trace the method to an earlier paper.

D. M. PAI