Sparse Norm Reflection Tomography for Handling Velocity Ambiguities

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Summary

Reflection seismology with the normal range of offsets encountered in seismic surveys is unable to resolve all the wavelength components of the subsurface velocity. One approach for overcoming this difficulty has been to drive the velocity determination towards models which make Geological sense.
The assumption behind this study is that the velocities within subsurface layers tend to be smooth, excluding isolated areas where they can vary discontinuously. For accommodating this assumption we use a tomographic velocity determination method which is based on a Cauchy sparse norm and a representation of the parameters in the wavelet transform domain.
The sparse tomography method is compared to conventional tomography in a synthetic example and a field data example. In general the sparse inversion produced superior results. In a second field data example, the new method is used for resolving artifacts caused by irregularities in the near surface as a result of permafrost.

Introduction

It has been pointed out by a number of investigators that subsurface velocity determination by reflection seismology suffers from ambiguities (Bickel, 1990, Bube et al., 1995, Tieman, 1994, Kosloff and Sudman, 2002). Bickel, 1990, and later Kosloff and Sudman, 2002, showed that even in the simplest case of a starting model containing a single uniform horizontal layer, there are velocity wave number components which cannot be resolved with the range of offsets normally used in reflection surveys. These ambiguities become larger in the multi-layer case, and with laterally varying velocity.
The velocity ambiguity cannot be reduced by increasing the density of the seismic data, or by performing velocity analysis at many locations. The strategy adopted by most tomographic inversion schemes is to add smoothness constraints into the functional to be minimized. While this approach stabilizes the solution, it only allows the determination of the very long wavelength components of the velocity variation.
An alternative approach for tackling the velocity ambiguity is to incorporate Geological assumptions into the tomographic inversion. Clapp and Biondi, 1999, used such an approach with grid tomography by designing a smoothing operator which acts primarily in the direction of the layering as opposed to perpendicular to the layers.
Sudman and Kosloff, 2002, presented a different idea where the tomographic equations are cast in the wavelet transform domain and a sparse Cauchy norm is used for the inversion. This study examines the latter approach and compares it to conventional tomography with synthetic and field data examples.
The Geological assumption behind our method is that normally velocities vary smoothly within layers, with the exclusion of isolated zones where they can vary abruptly, such as for example in over-pressured zones or in permafrost. However this notion does not necessarily translate into sparseness in ordinary tomography. For example, a constant velocity update within a layer means that all interpolation points within the layer are equally updated. However, in the wavelet domain, a constant update is represented by a single parameter which corresponds to a dc shift. In general the Geological assumption corresponds to a sparse representation in the wavelet domain, where only the parameters around the locations of the discontinuities receive updates.

Tomography in the Wavelet Domain

Consider a 2D subsurface model with N_x CMP stations in the horizontal direction, and N_L layers. The tomography updates the values of the velocity and layer depth (actually the equivalent parameters of slowness and vertical time as in Kosloff et al., 1996) at all CMP locations and all layers. The input for the tomography, are time errors at all CMP locations and layers for all the offsets used in the survey. The discretized tomography equations can be written as,

$$A\delta m = \delta t$$

where A , is the influence matrix, δm is a vector of size $2\times N_x \times N_L$ containing the slowness and vertical time updates at all layers and CMP locations. δt is a vector containing the time errors at all locations and offsets. In conventional tomography, the parameter space is reduced by interpolating the updates from a sparse grid of values at interpolation points (e.g Kosloff et al., 1996).
In the wavelet representation, the unknowns become, $\delta m' = W\delta m$ instead of δm, where W represents the layer by layer wavelet transform operator. The modified tomography equations thus become,

$$AW^{-1}\delta m' = \delta t\ .$$

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Operationally, this requires the calculation of the entries of the influence matrix for every CMP-layer-offset data value, followed by a layer by layer wavelet transform on its entries with the operator W^{-T}. For this study we used a bi-orthogonal spline FIR wavelet transform with four coefficients (Averbuch and Zheludev, 2004). This wavelet basis has symmetrical filters with a good compromise between spatial and wave number localization.
After recasting the tomography equations in the wavelet domain, the resulting over- determined system can be solved by standard techniques. As stated previously, the present study used sparse inversion based on the Cauchy norm (Sacchi et al., 1998).

Synthetic Example

The synthetic model consisted of three horizontal layers with respective interface depths of 1000m, 2000m, and 3000m. The velocities within the layers had abrupt discontinuities (Fig 1). The CMP spacing in this model was 25m, and there were 50 offsets with an offset interval of 50m.
For the test, we used an initial model with constant layer velocities of 1925m/sec, 2050m/sec, and 2350m/sec respectively. Fig 2 shows the result of migration with a velocity section derived from the initial model. The figure shows that the use of incorrect velocities produced non horizontal interfaces.
Figure 3 shows the velocities which were reproduced after two iterations of migration and sparse tomography. This figure shows that the sparse tomography was able to recover the true velocity quite well.
For comparison, Fig 4 presents the velocities which were reproduced after two iterations of conventional tomography. This figure shows that the main trend of the true velocity was reproduced, however the velocities in the figure are highly smoothed without a good indication of the sharp discontinuities in the model.

North Sea Field Data Example

This example presents results of the first stage of velocity determination where the inversion is carried out for the uppermost layers. Fig 5 shows a depth section which was obtained with the initial velocity. This section also shows the three layers of the depth model. In this example the CMP spacing was 12.5m. and there were 60 offsets with an offset spacing of 50m. The initial model layer velocities had constant values of 1500m/sec, 1900m/sec, and 3450 m/sec respectively.

Fig 6 and Fig 7 show the tomographically determined layer velocities after a single iteration of sparse inversion and conventional tomography respectively. The interesting observation from the sparse inversion (Fig 6) is that there is an indication of different constant velocities for the third layer (colored in brown) on the two sides of the salt body. There is a hint of such a variation in the velocities derived from conventional tomography but the interpretation is more ambiguous.

Permafrost Example

This example demonstrates the use of tomography for resolving near surface irregularities. Fig 8 shows a depth section which was obtained in an area containing permafrost. The CMP spacing in this example was 25m, and there were 26 offsets with an increment of 120m. The figure was obtained with a simple laterally uniform velocity section. Figure 8 clearly shows the effect of the near surface which causes irregular undulations in the deeper horizons.
In order to reduce the effect of the near surface, we introduced a single horizontal fictitious layer at the top of the model and solved for its velocity while maintaining its thickness constant. The input data for the inversion were the reflections from the deeper horizons of the model. Since such a layer does not necessarily match in thickness the real permafrost layer, the derived velocities there may not correspond to realistic values. However, the time delays produced by this layer should reduce the irregularities caused by the near surface anomalies in the deeper reflection horizons.
Fig 9 shows the depth section obtained with the near surface velocity derived by the tomography using sparse inversion. The conventional tomography did not work well for this example and the results obtained by it are not shown.. Compared to Fig 8, this figure shows a definite improvement in the continuity of the horizons. Fig 10 shows the tomographically determined velocity for the near surface layer. The velocity variation in this figure is quite irregular.

Conclusions

Due to a fundamental ambiguity, there can be different subsurface velocity models which equally explain the reflection data. This study has shown that sparse tomography may offer improvements over conventional tomography especially in structures where there are rapid velocity changes. In general the velocities derived by the sparse inversion are simpler and with less undulations, and hence make more geological sense.
The approach used in this study may have further benefits in deriving anisotropic parameters. In anisotropic inversion the ambiguity in the values of the material parameters is much greater than in isotropic inversion and hence the idea of driving the solutions towards simpler models appears reasonable.

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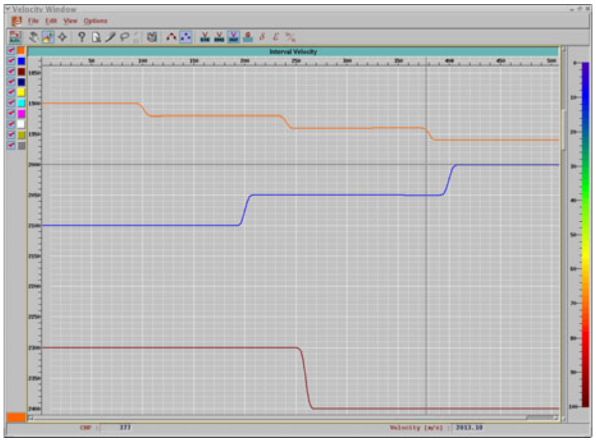
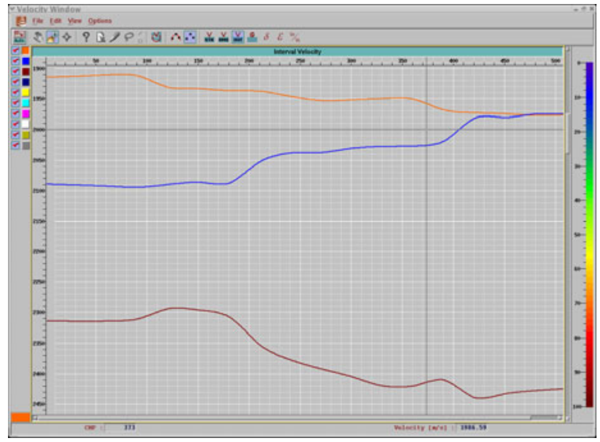


Fig 1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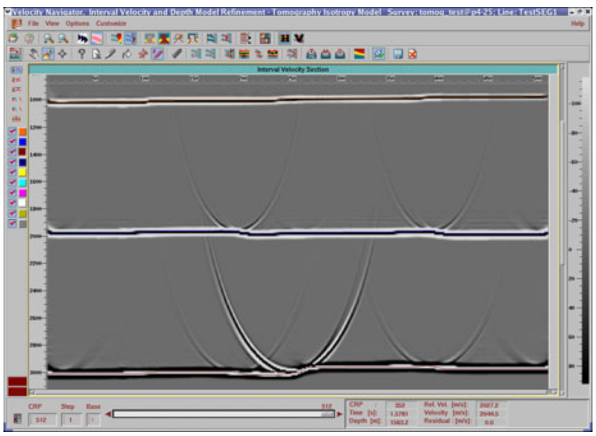
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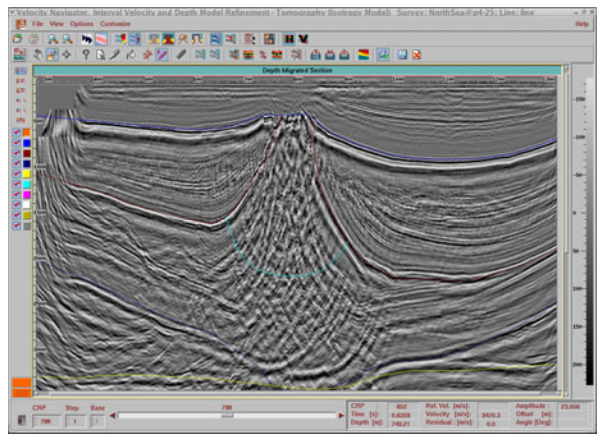
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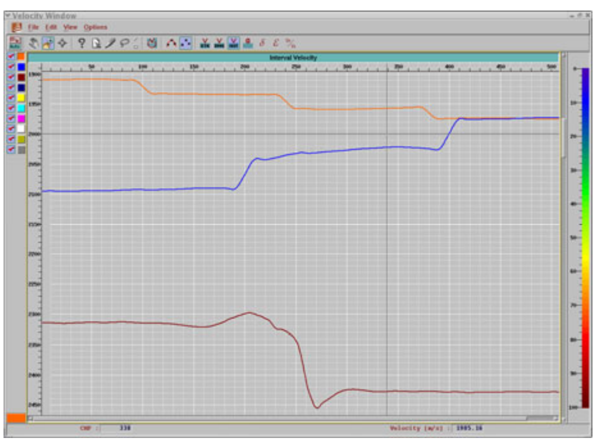
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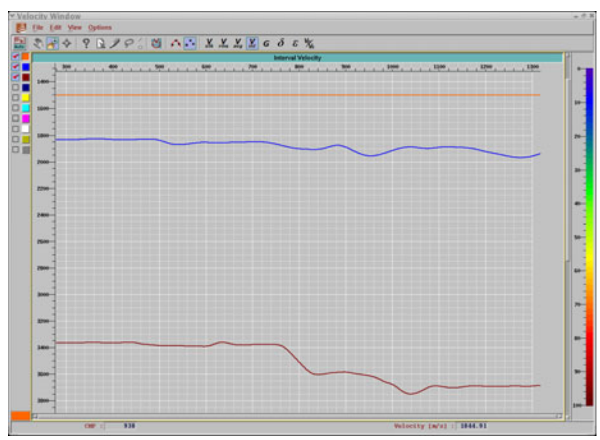
Fig 6

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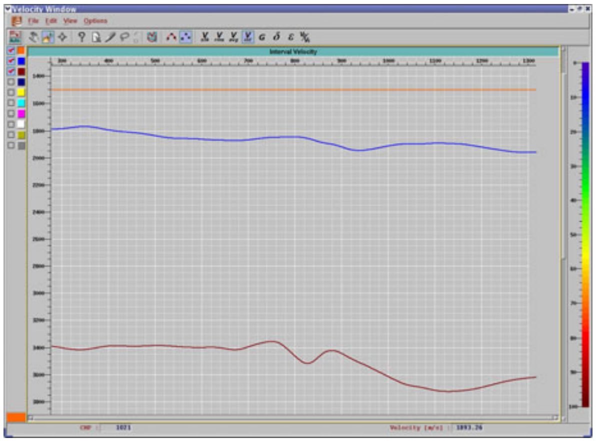
Fig 7

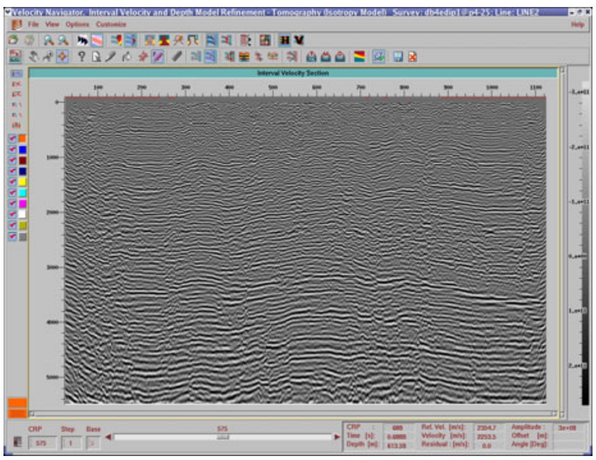
Fig 8

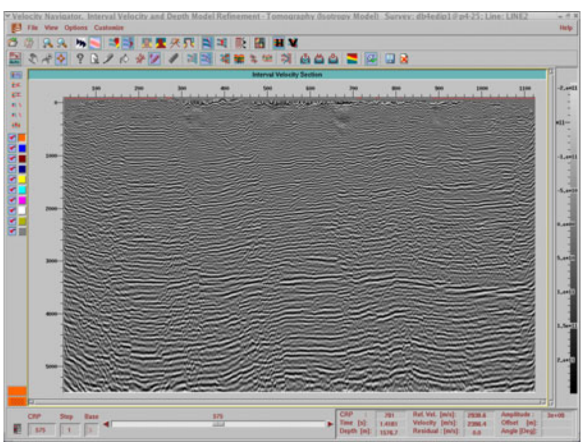
Fig 9

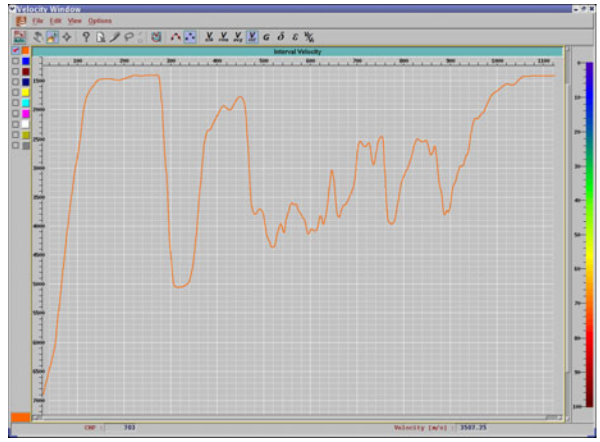
Fig 10

References

Averbuch, A., and Zheludev, V., 2004. A new family of spline based biorthogonal wavelet transforms and their application to image compression. IEEE Trans. Image Proc. 13, 7, 993-1007.

Bickel, S.H., 1990, Velocity-depth ambiguity of reflection travel times: Geophysics, 718-729.

Bube, K.P., Langan, R.T, and Resnick, R.R., 1995, Theoretical and numerical issues in the determination of reflector depths in seismic reflection tomography: JGR No B7 12449-12458.

Clap, R., and Biondi, B., 1999. Preconditioning tau tomography with geologic constraints. Sep-100, 35-50.

Kosloff, D., and Sudman, Y., 2002. Uncertainty in determining interval velocities from surface reflection data. Geophysics 67, 3, 952-963.

Sudman, Y., and Kosloff, D., 2002. High resolution travel time tomography. SEG-Salt Lake City annual meeting abstracts.

Tieman, H., J., 1994. Investigating the depth-velocity ambiguity of reflection travel times: Geophysics 59, 1763-1773.

Sacchi, M. Ulrich, T., and Walker, C., Interpolation and extrapolation using a High-Resolution Discrete Fourier Transform. IEEE Trans. Signal Proc. 46, 1, 31-38.

EDITED REFERENCES

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References

Averbuch, A., and V. Zheludev, 2004, A new family of spline based biorthogonal wavelet transforms and their application to image compression: IEEE Transactions on Image Processing, **13**, 993-1007.

Bickel, S. H., 1990, Velocity-depth ambiguity of reflection travel times: Geophysics, **55**, 718-729.

Bube, K. P., R. T. Langan, and R. R. Resnick, 1995, Theoretical and numerical issues in the determination of reflector depths in seismic reflection tomography: Journal of Geophysical Research, no B7 12449-12458.

Clap, R., and B. Biondi, 1999, Preconditioning tau tomography with geologic constraints: Stanford Exploration Project, **100**, 35-50.

Kosloff, D., and Y. Sudman, 2002, Uncertainty in determining interval velocities from surface reflection data: Geophysics, **67**, 952-963.

Sudman, Y., and D. Kosloff, 2002, High resolution travel time tomography: 72nd Annual International Meeting, SEG, Expanded Abstracts, page.

Tieman, H. J., 1994. Investigating the depth-velocity ambiguity of reflection travel times: Geophysics, **59**, 1763-1773.

Sacchi, M., T. Ulrich, and C. Walker, 1998, Interpolation and extrapolation using a High-Resolution Discrete Fourier Transform: IEEE Transactions on Signal Processing, **46**, 31-38.