Evaluation of Multicomponent Depth Migration

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Summary

This paper presents an evaluation of multi component depth migration. This type of migration can be carried out by most techniques commonly used for conventional single component acoustic migration. These can be broadly categorized as downward continuation methods and reverse time migration methods. We demonstrate downward continuation by extensions of the phase shift method introduced by Gazdag (1978) and Bolodni et al. (1978), as well as by the generalized phase shift method (Kosloff and Kessler, 1986). We then evaluate elastic reverse time migration (Sun and McMechan, 1986, Chang and McMechan, 1987).

This study indicates that although multi component elastic migration can be carried out technically, however from a practical point of view this may be using the migration concept beyond its applicable limits. Specifically we show that when the correct velocity is used reverse time migration is incapable of reproducing correct amplitudes. Furthermore, use of the exact velocity and the free surface boundary condition creates false events in all methods. Consequently, smoothed velocities or a modification of the boundary conditions are needed thus abandoning hope of reproducing correct amplitudes. It may finally turn out that simple acoustic migration of P and S potentials is the most promising alternative. We present a series of tests on simple synthetic examples which supports this opinion.

Migration By Downward Continuation

The migration is based on solution in depth of a system of equations containing the displacements and vertical tractions as unknowns. The system is derived from the equations of momentum conservation and the stress-strain relation for an isotropic solid. The temporally transformed equations of momentum conservation are given by:

$$\frac{\partial \tilde{\sigma}_{xx}}{\partial x} + \frac{\partial \tilde{\sigma}_{xz}}{\partial z} = -\rho\omega^2 \tilde{u}_x \qquad (1)$$

$$\frac{\partial \tilde{\sigma}_{xz}}{\partial x} + \frac{\partial \tilde{\sigma}_{zz}}{\partial z} = -\rho\omega^2 \tilde{u}_z$$

where x and z are respectively the horizontal and vertical coordinates, ω is the temporal frequency, ρ is the density , $\tilde{\sigma}_{xx}$, $\tilde{\sigma}_{xz}$ and $\tilde{\sigma}_{zz}$ are the transformed stresses, and $\tilde{u}_x$ and $\tilde{u}_z$ are the horizontal and vertical displacements respectively.

The stress displacement relations are given by,

$$\tilde{\sigma}_{xx} = (\lambda + 2\,\mu)\frac{\partial \tilde{u}_x}{\partial x} + \lambda\frac{\partial \tilde{u}_z}{\partial z}$$

$$\tilde{\sigma}_{zz} = \lambda\frac{\partial \tilde{u}_x}{\partial x} + (\lambda + 2\,\mu)\frac{\partial \tilde{u}_z}{\partial z} \qquad (2)$$

$$\tilde{\sigma}_{xz} = \mu\left(\frac{\partial \tilde{u}_x}{\partial z} + \frac{\partial \tilde{u}_z}{\partial x}\right),$$

where λ and μ are respectively the rigidity and the shear modulus.

After the elimination of $\tilde{\sigma}_{xx}$ from (1) and (2) the equations can be written as a system of the form,

$$\frac{d\mathbf{U}}{dz} = A\mathbf{U} \quad , \qquad (3)$$

where $\mathbf{U}^T = (\tilde{u}_x\,,\ \tilde{u}_z\,,\ \tilde{\sigma}_{xz}\,,\ \tilde{\sigma}_{zz})$ is the motion-stress vector, and

$$A = \begin{pmatrix} 0 & -\frac{\partial}{\partial x} & \frac{1}{\mu} & 0 \\ \frac{\lambda}{(\lambda+2\mu)}\frac{\partial}{\partial x} & 0 & 0 & \frac{1}{(\lambda+2\mu)} \\ -\omega^2\rho - 4\alpha & 0 & 0 & -\frac{\partial}{\partial x}\left(\frac{\lambda}{\lambda+2\mu}\right) \\ 0 & -\omega^2\rho & -\frac{\partial}{\partial x} & 0 \end{pmatrix}$$

where $\alpha = \frac{\partial}{\partial x}\left(\frac{\mu\lambda + \mu^2}{\lambda + 2\mu}\right)\frac{\partial}{\partial x}$. The downward continuation consists of the solution of (3) in depth z.

Solution By The Generalized Phase Shift Method

After a discretization in the horizontal coordinate and specification of a horizontal derivative approximation, equation (3) becomes a system of $4N_x$ coupled equations where N_x denotes the number of seismic traces. Assuming A is constant between z and $z + dz$ (but can be laterally variant) the formal solution to (1) is given by,

$$\mathbf{U}(z+dz) = \exp(A\ dz)\,\mathbf{U}(z) \quad . \qquad (4)$$

The generalized phase shift method consists of an expansion of (4) given by,

$$\mathbf{U}(z+dz) = \sum_{k=0}^{M} C_k \, J_k(R\,dz)\, Q_k(\frac{A}{R})\, \mathbf{U}(z) \quad , \qquad (5)$$

where $C_0 = 1$ and $C_k = 2$ for $k \neq 2$, J_k is the k^{th} order Bessel function and $Q_k \mathbf{U}_z$ are vectors which are generated recursively according to,

$$Q_0\, \mathbf{U}(z) = \mathbf{U}(z),$$

$$Q_1(\frac{A}{R})\mathbf{U}(z) = A\, \mathbf{U}(z)/\, R$$

and,

$$Q_{k+1}\, (\frac{A}{R})\mathbf{U}(z) = Q_{k-1}\, (\frac{A}{R})\mathbf{U}(z) + 2\frac{A\,dz}{R}\, Q_k(\frac{A}{R})\, \mathbf{U}(z) \ .$$

R should be chosen larger than the range of eigenvalues of A, which based on the uniform velocity case is approximately $\pi/\, V_{s_{\min}}$, where $V_{s_{\min}}$ is the lowest S wave velocity value between depths z and $z + dz$.

The number of terms required in the summation (5) is slightly larger than Rdz (Kosloff and Kessler,1986).

Solution By The Phase Shift Method

When the velocities are laterally uniform the equations can be Fourier transformed over the x coordinate. For each value of the horizontal wavenumber k equation (3) becomes a set of four coupled ordinary differential equations, the solution of which is given by,

$$\mathbf{U}(z+dz) = exp(A\ dz\ )\ \mathbf{U}(z) \quad . \qquad (6)$$

Here $\exp(A\ dz) = P\ (z,z+dz)$ is the well known propagator matrix whose entries can be calculated explicitly (Aki and Richards, 1980). The downward continuation is thus carried out by a series of multiplications by 4x4 propagator matrices.

The type of phase shift migration introduced here includes both upgoing and downgoing waves and implicitly contains the correct transmission coefficient between layers. This is in contrast to the phase shift method in Gazdag (1978) which only uses upgoing waves. Seemingly this should result in more accurate amplitudes, however it will be shown that this is not always the case.

Elimination Of Evanescent Components

The evanescent components correspond to the real eigenvalues of the operator $A\ dz$. As in the acoustic case they can cause numerical instability in the form of growing exponential solutions and therefore need to be eliminated. For the elimination we chose the criterion $|k| < \frac{\omega}{V_{\max}}$ where $V_{\max}$ is the highest velocity in a given layer (Kosloff and Baysal,1983). Although this criterion has proven stable, it may cause loss of steeply dipping events for S waves and also for P waves in low velocity regions.

Surface Boundary Condition

From a physical point of view the tractions $\tilde{\sigma}_{xz}$ and $\tilde{\sigma}_{zz}$ must vanish on the surface $z = 0$. However this implies that both upgoing and downgoing waves exist from the onset. We show through examples that the downgoing waves can convert to upgoing waves upon crossing sharp interfaces and create false events at locations where reflectors do not exist. It thus appears that either a form of velocity smoothing is necessary or else the free surface condition needs to be replaced by another condition, like for example the condition of existence of only upgoing energy.

Potential Separation

The displacement field can be obtained from P and S potentials $\tilde{\phi}$ and $\tilde{\psi}$ respectively according to,

$$\tilde{\mathbf{u}}_x = \frac{\partial \tilde{\phi}}{\partial x} - \frac{\partial \tilde{\psi}}{\partial z} \qquad (7a)$$

and,

$$\tilde{\mathbf{u}}_z = \frac{\partial \tilde{\phi}}{\partial z} + \frac{\partial \tilde{\psi}}{\partial x} \quad . \qquad (7b)$$

Assuming that the the velocity on the surface is constant (not always a good assumption) equations (7) can be Fourier transformed with respect to x to yield,

$$\tilde{\tilde{\mathbf{u}}}_x = ik\tilde{\tilde{\phi}} - \frac{\partial \tilde{\tilde{\psi}}}{\partial z}$$

and,

$$\tilde{\tilde{\mathbf{u}}}_z = \frac{\partial \tilde{\tilde{\phi}}}{\partial z} + ik\tilde{\tilde{\psi}}$$

with $i = \sqrt{-1}$.

This type of separation allows calculation of $\tilde{\tilde{\phi}}$ and $\tilde{\tilde{\psi}}$ from $\tilde{\tilde{\mathbf{u}}}_x$ and $\tilde{\tilde{\mathbf{u}}}_z$ based on the assumed surface boundary condition of the migration. For example it can be shown that the assumption that all recorded energy is upgoing leads to,

$$\tilde{\tilde{\phi}} = \frac{-ik\tilde{\tilde{u}}_x - i\eta_s \tilde{\tilde{u}}_z}{k^2 + \eta_p \eta_s} \qquad (8)$$

and,

$$\tilde{\tilde{\psi}} = \frac{i\eta_p \tilde{\tilde{u}}_x - ik\tilde{\tilde{u}}_z}{k^2 + \eta_p \eta_s} \quad ,$$

where $\eta_s = \left[\frac{w^2}{V_s^2} - k^2\right]^{1/2}$ and $\eta_p = \left[\frac{w^2}{V_p^2} - k^2\right]^{1/2}$ with V_s and V_p the S and P velocities respectively.

Migration Of Potentials

Once $\tilde{\phi}$ and $\tilde{\tilde{\psi}}$ are obtained by (8) or an equivalent equation for another surface boundary condition, $\tilde{\phi}$ and $\tilde{\psi}$ can be calculated by an inverse Fourier transform. The

potentials can then be migrated by ordinary acoustic migration. This migration obviously does not yield accurate reflection or transmission amplitudes, however it does give correct arrival times for pure P and S wave paths. It therefore appears that with smoothed velocities the errors in this type of migration will not be very different from those in single component acoustic migration. However there is the advantage of obtaining independent P and S sections where errors in one of the velocity types will not affect the section of the other type.

Reverse Time Migration

Elastic reverse time migration consists of solving the equations of dynamic elasticity where the time reversed shot record serves as the surface boundary condition (McMechan,1983, Baysal et. al,, 1983). This migration which is formulated in the x-z-t-domain has the advantage of not having to deal with evanescent energy. However, with the accurate interval velocities it is incapable of reproducing correct amplitudes since by propagating all wave fronts backwards one does not obtain the original wave amplitude which emanated from the shot. As tests on synthetic data verify the problem of creating false events is more serious in reverse time migration than with downward continuation techniques. It therefore appears that sharp velocity contrasts need to be avoided or at least all interfaces must be approximately impedance matched. However in this case it is questionable whether use of this expensive migration is justified.

Conclusion

Although technically multicomponent migration can be carried out without difficulty, from a practical viewpoint one must be aware of the limitations of the migration concept itself. Thus even when using correct interval velocities it does not appear that multicomponent migration can reproduce correct amplitudes or equivalently impedance contrasts. However when used with caution multicomponent migration has the advantage of sensitivity and thus

ability to evaluate both P and S velocities. As it appears that the objective of producing correct amplitudes needs to be abandoned, perhaps the most promising strategy is to affect multicomponent migration by separate migration of P and S potentials.

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