behavior and yield large reflected amplitudes which contaminate the seismic response.

Lysmer and Kuhlenmeyer (1969) introduced a method which is based on visco-elastic boundary conditions; thereby compressional waves will be attenuated whereas shear waves are not affected comparably. Smith (1974) presented a method which subjects the displacement field at the boundaries to Dirichlet and Neumann conditions, respectively. In this way a superposition of the corresponding numerical solutions totally cancels the reflected wave field. Since this method requires 2^n (n = number of boundaries) repetitions of the computation large CPU-times result. In 1977, Engquist and Majda and Clayton and Engquist published a family of absorbing boundary conditions which represent first and second order one-way approximations of the equations of motion for acoustic and elastic waves. Similar ideas were followed by Reynolds (1978) who introduced one-way approximations by factorization of the equations of motion in terms of the incident and reflected part of the wave field.

We tested both methods for elastic waves and found that they work excellent for angles of incidence $\phi < 50°$ whereas for larger angles residual reflections become significant. To reduce the amplitudes of these residual reflections, we suggest proceeding in the following way: (1) Implementation of Clayton and Engquist's second order absorbing boundary conditions into the FD-algorithm. (2) Introduction of a visco-elastic bordering strip which additionally reduces boundary reflection by dissipation.

To introduce dissipation into the FD algorithm, we make use of a complex angular frequency $\tilde{\omega} = \omega - i\tau$ which leads to an attenuation of the displacement field by a factor $e^{-\tau t}$. Since this is equivalent to complex seismic velocities α, β for P- and S-waves, respectively, an additional interface has to be considered generating reflected P- and SV-waves. For $\tau/\omega = .05$ the reflection and transmission coefficients for incident plane harmonic P- and SV-waves, respectively, are shown in Figures 1 and 2.

Incident P-wave (Figure 1). The coefficients $|\acute{P}\acute{P}|$, $|\acute{P}\acute{S}|$, and $|\grave{P}\acute{S}|$ do not become significant for practical calculations, i.e., the incident P-wave passes the interface without a relevant loss of energy and propagates mainly as a P-wave in the visco-elastic medium.

Incident SV-wave (Figure 2). In this case the reflected P-wave can be expected to contaminate the seismic response for angles $\phi > 40°$. If, however, SV-waves are assumed to be generated only from first order discontinuities, the influence of the reflected P-wave on the seismic response should not become too large. The coefficients $|\grave{S}\acute{S}|$ and $|\grave{S}\grave{S}|$ are seen to be less than 1 p.c. of the incident amplitude and do not affect the numerical calculations. Finally, in the visco-elastic zone the converted P-wave becomes dominant.

In the following numerical calculation we tested the method. We model the propagation of a compressional P-wave in a structure with a horizontal first order discontinuity (Figure 3, horizontal heavy line) separating two homogeneous elastic media. In the upper medium we chose α = 2.0 km/s and in the lower half-space α = 4.0 km/s. In both media we put the density $\rho = 1$ and the shear wave velocity $\beta = \alpha/\sqrt{3}$. At the boundaries of the model we used second order paraxial approximations of Clayton and Engquist and inserted a visco-elastic strip at the left and right side of the model (Figure 3, vertical heavy lines). The strip has a width of 18 spatial increments. Figure 3a shows the horizontal (H) and vertical (V) component of the direct P-wave. In Figure 3b the components of the reflected wave field (PP, $\acute{P}SV$) can be seen. In this case after applying the Clayton and Engquist algorithm residual reflections can be observed which are not further damped by the visco-elastic strip. Figure 3c finally shows the result of additional damping which effectively reduces the residual boundary reflections.

A Nonreflecting Boundary Condition for Discrete Acoustic and Elastic Wave Calculations S15.2

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One of the nagging problems which appears in the application of discrete solution methods to wave propagation problems is the presence of reflections or wraparound from the boundaries of the numerical mesh. In this paper we describe a scheme for the elimination of these unwanted events which can be applied to a wide class of wave equations and numerical methods. The scheme is first presented as an empirical approach based on a gradual elimination of wave amplitudes along the bondaries of the numerical grid. However, in order to apply the method to implicit and semi-implicit schemes which do not use explicit time stepping, the absorbing boundary condition is rederived and cast in the form of a modified wave equation. This derivation gives the additional benefit of the ability to evaluate the effectiveness of the absorbing boundary a priori, and adjust its parameters without having to make costly computer runs.

The absorbing boundary condition is demonstrated with examples from acoustic and elastic wave propagation with the Fourier solution method.

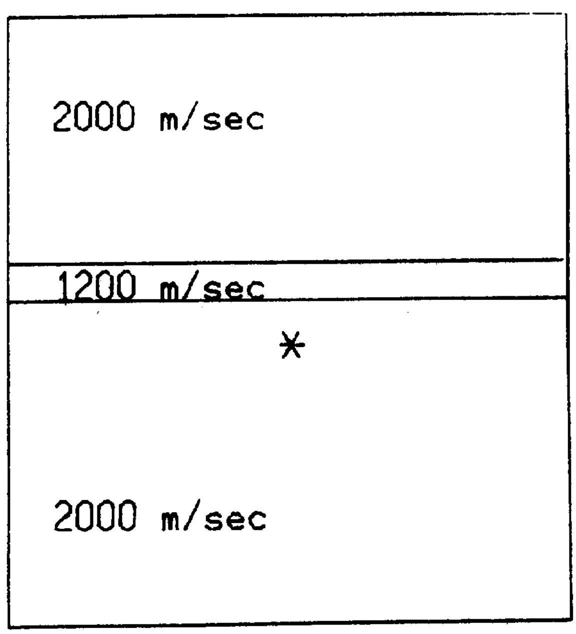

FIG. 1.

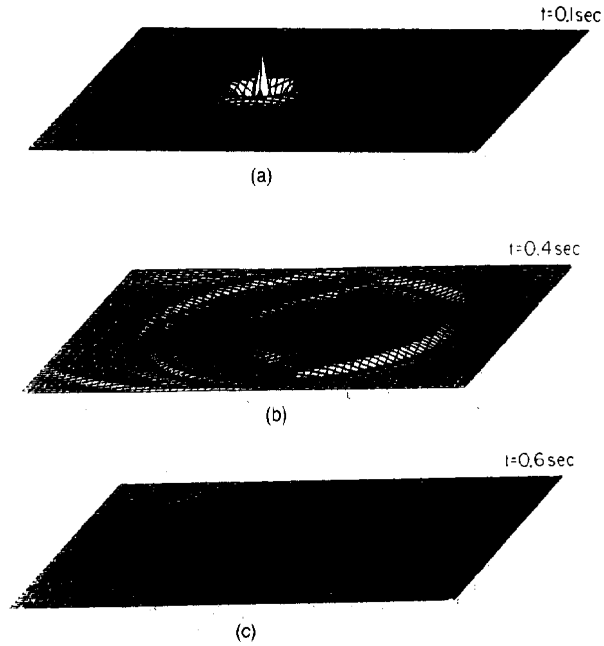

FIG. 2.

The empirical approach

For illustration we demonstrate the nonreflecting boundary condition for the Fourier method solution of the acoustic wave equation (Kosloff and Baysal, 1982).

Let $p^n(i\,j)$ denote the pressure at time $t = nDt$ and at spatial location $X = iDx$, $y = jDy$, $i = 1 \ldots, Nx$, $j = 1 \ldots, Ny$, with dt denoting the time step size and Dx and Dy denoting the mesh size in the x and y directions. A typical time step of the solution method runs as follows, (Kosloff and Baysal, 1982). (a) Calculate

$$R^n(i\,j) = \frac{\partial^2 p^n}{\partial x^2} + \frac{\partial^2 p^n}{\partial y^2}$$

by the Fourier approximation. (b) Integrate in time according to

$$\dot{p}^{n+1/2}(i\,j) = \dot{p}^{n-1/2}(i\,j) + Dt.\ R^n(i\,j).\ C^2(i\,j),$$

and

$$p^{n+1}(i\,j) = p^n(i\,j) + Dt\,\dot{p}^{n+1/2}(i\,j),$$

with $C(i\,j)$ denoting the acoustic velocity. The calculations in (a) and (b) are repeated for the desired number of time steps.

For the nonreflecting boundary condition the values of p^{n+1} and $\dot{p}^{n+1/2}$ are slightly reduced after each time step in a strip of nodes surrounding the numerical mesh. For the Fourier method we found a strip width of 20 nodes sufficient to reduce side reflections to a few percent. The amplitude reduction in each strip is gradually tapered from a zero value in the interior boundary of the strip.

Modified acoustic wave equation

For derivation of the modified acoustic wave equation, it is convenient to consider the wave equation as a set of coupled first-order equations given by:

$$\frac{\partial}{\partial t}\begin{bmatrix} P \\ V \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ C^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) & 0 \end{bmatrix}\begin{bmatrix} P \\ \bar{V} \end{bmatrix},$$

where V is equal to the pressure time derivative. The amplitude reduction step of the previous section can then be considered as a first-order time stepping scheme for the equation:

$$\frac{\partial}{\partial t}\begin{bmatrix} P \\ V \end{bmatrix} = \begin{bmatrix} -\alpha & 0 \\ 0 & -\alpha \end{bmatrix}\begin{bmatrix} P \\ V \end{bmatrix},$$

where α is the reduction factor which attains its largest value at the grid boundaries. The numerical scheme can now be viewed as a splitting method of a single system of equation given by:

$$\frac{\partial}{\partial t}\begin{bmatrix} P \\ V \end{bmatrix} = \begin{bmatrix} -\alpha & 1 \\ C^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) & -\alpha \end{bmatrix}\begin{bmatrix} P \\ V \end{bmatrix}.$$

This system can be integrated numerically as in the example of the next section. Most integration schemes for the acoustic wave equation, however, are based on a single second-order wave equation. An equation of this type is achieved after the elimination of V from the previous equation. This gives,

$$\frac{\partial^2}{\partial t^2} = C^2\left(\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2}\right) - 2\alpha\frac{\partial p}{\partial t} - \alpha^2 p.$$

This equation, like the exact acoustic wave equation, can be integrated in time after a choice of a stable integration scheme. In addition, this equation can be solved analytically for one space dimension by the propagator matrix method and the effectiveness of the absorbing regions for different values of the parameter α can be assessed.

Example: Acoustic wave propagation

We consider acoustic wave propagation in a region containing a low velocity region (Figure 1). A point source was excited in the high velocity region slightly below the layer. The amplitudes of successive times were calculated with the Fourier method (Kosloff and Baysal, 1982) with a time integration scheme based on the semi implicit method of Tal Ezer (1984).

Figure 2a–c presents wave amplitudes at progressive times with inclusion of absorbing boundaries. As the figures show, wraparound or boundary reflections have been virtually eliminated. For comparison Figure 3 shows a corresponding snapshot which was obtained without the absorbing boundary condition.

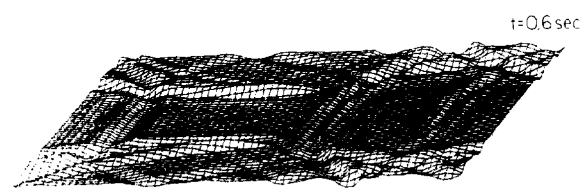

FIG. 3.

References

Kosloff, D., and Baysal, E., 1982, Forward modeling by a Fourier method: Geophysics, **47**, 1402–1412.